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A FORTRAN Computer Program for Calculating The Propagation of Plane, Cylindrical, or Spherical Finite Amplitude Waves

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A numerical solution to the generalized Burgers' radial wave equation has been developed; it allows one to calculate stepwise the harmonic content of a finite amplitude wave in the frequency domain for the case of plane, cylindrical, or spherical geometry. The finite amplitude wave may have any initial harmonic content with arbitrary phase, and the absorption coefficient of each harmonic is independently adjustable. Remaining in the frequency domain allows a much larger step than conventional algorithms, which alternate between the time and frequency domains. A listing of the computer program is included.

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CONTENTS

A FORTRAN COMPUTER PROGRAM FOR CALCULATING
THE PROPAGATION OF PLANE, CYLINDRICAL, OR SPHERICAL
FINITE AMPLITUDE WAVES

INTRODUCTION

Several adequate theories [1,2,3], based on approximations to the
nonlinear wave equation, have been developed to describe the behavior of a
one-dimensional wave of moderate amplitude as it propagates through a nonlinear
fluid. Most of these theories, however, are not conveniently applied to the
problem of describing the propagation when the wave is of arbitrary initial
waveform or when the linear absorption has an arbitrary frequency dependence.
To handle these more general cases, investigators [4,5,6] have adapted the
phenomenological model of Fox and Wallace [7] to use a high-speed computer
to calculate the propagation stepwise. The distance of propagation in this
model is divided into small intervals. The wave is first allowed to distort
over one interval and is then corrected to account for absorption and geometrical
spreading. The procedure is then repeated for the new waveform over the next
interval. The use of small intervals preserves the interaction between the
distortion, the absorption, and the geometrical spreading mechanisms. Since
the distortion mechanism is applied in the particle velocity domain with
absorption and geometrical spreading being applied in the frequency domain,
it is necessary to switch back and forth between the two domains during each
step. Even with the use of the Fast Fourier Transform (FFT), this procedure
is a time-consuming process. In addition, one must take special care in
applying the distortion mechanism when the waveform has a very steep shock-like
portion.

We describe in this report a new procedure for calculating the
propagation of plane, cylindrical, or spherical finite amplitude waves.
This procedure performs the stepwise calculations entirely in the frequency
domain, thus avoiding both the use of the FFT and the steep waveform problems.
We also describe a FORTRAN computer program called FAW that implements the
new procedure.

The theory behind the procedure is described in Sec. I. A discussion
of the relative importance of linear absorption and nonlinear effects is
presented in Sec. II. In Sec. III the numerical implementation of the procedure

is developed. This is followed in Sec. IV by a description of the computer
program FAW. Included are discussions of the significant FORTRAN variable
names, the major computational blocks, parameter input, printed output,
number of harmonics required in the calculation, and initial step size.
Section V contains a comparison of results obtained using the new procedure
with results obtained using the phenomenological model. The report concludes
with appendices containing sample output and a program listing of FAW.

I. THEORY

An approximate nonlinear wave equation valid for one-dimensional and
progressive plane, cylindrical, and spherical waves in a lossless fluid
is the generalized Burgers' equation [8]

$$\frac{\partial U}{\partial r} + (a/r)U - bU\frac{\partial U}{\partial \tau} = 0, \tag{1}$$

where U = particle velocity,

r = spatial coordinate,

τ = retarded time $(t - \frac{r}{c_o})$,

c_o = small signal sound speed,

$b = \beta/c_o^2$

$\beta = (1 + \frac{B}{2A})$, the nonlinearity parameter, and

a = 0 (plane waves),

= 1/2 (cylindrical waves), or

= 1 (spherical waves).

Equation (1) is modified to include linear absorption by noting that in the
absence of nonlinearity the amplitude decays according to

$$U(r) = U_o (r_o/r)^a \exp [- \alpha(\omega)(r-r_o)] \tag{2}$$

or

$$\frac{\partial U}{\partial r} = -(a/r) U - \alpha(\omega) U. \tag{3}$$

2

The first term on the right-hand side of Eq. (3) represents the loss due to geometrical spreading and is already included in Eq. (1). The second term represents the loss due to linear absorption. Adding this term to Eq. (1) results in the following nonlinear equation for a lossy fluid with arbitrary frequency-dependent absorption where dispersion has been neglected:

$$\frac{\partial U}{\partial r} + (a/r)U - bU\,(\partial U/\partial \tau) = -\,\alpha(\omega)\,U. \tag{4}$$
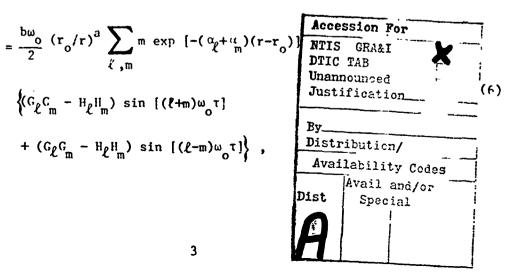
We now choose as a trial solution a Fourier series of linear damped waves of arbitrary phase with amplitudes that are a function of the spatial coordinate r:

$$U(r,\tau) = \sum_{k=1}^{\infty} (r_o/r)^a \left\{ G_k \sin(k\omega_o\tau) + H_k \cos(k\omega_o\tau) \right\} \exp[-\,\alpha_k(r-r_o)], \tag{5}$$

where α_k is the absorption coefficient appropriate for the kth harmonic. The fundamental frequency ω_o is chosen less than or equal to $1/\tau_o$, where τ_o is either the period of the initial waveform at $r=r_o$ when the waveform is periodic or it is a time length sufficiently long to contain the resulting waveform at all desired distances when the waveform is transient. Substitution of Eq. (5) into Eq. (4) yields two coupled differential equations governing the behavior of the amplitude components G_k and H_k as a function of the spatial coordinate r:

$$\sum_{k=1}^{\infty} \frac{\partial G_k}{\partial r} \sin(k\omega_o\tau)\exp[-\alpha_k(r-r_o)]$$

$$= \frac{b\omega_o}{2} (r_o/r)^a \sum_{\ell,m} m \exp[-(\alpha_\ell+\alpha_m)(r-r_o)]$$

$$\left\{ (G_\ell G_m - H_\ell H_m)\sin[(\ell+m)\omega_o\tau] \right.$$

$$\left. + (G_\ell G_m - H_\ell H_m)\sin[(\ell-m)\omega_o\tau] \right\}, \tag{6}$$

3

and

$$\sum_{k=1}^{\infty} \frac{\partial H_k}{\partial r} \cos (k\omega_o \tau) \exp [-\alpha_k(r-r_o)]$$

$$= \frac{b\omega_o}{2} (r_o/r)^a \sum_{\ell,m} m \exp [-(\alpha_\ell + \alpha_m)(r-r_o)] \qquad (7)$$

$$\left\{ (H_\ell G_m + G_\ell H_m) \cos [(\ell+m)\omega_o \tau] \right.$$

$$\left. + (H_\ell G_m - G_\ell H_m) \cos[(\ell-m)\omega_o \tau] \right\}.$$

Factoring out terms of the same frequency (i.e., terms in which $\ell+m = k$, $\ell-m = k$, and $\ell-m = -k$) results in

$$\frac{\partial G_k}{\partial r} = \frac{b\omega_o}{2} (r_o/r)^a \left\{ \sum_{m=1}^{k-1} m(G_{k-m}G_m - H_{k-m}H_m) \exp [-(\alpha_{k-m} + \alpha_m - \alpha_k)(r-r_o)] \right.$$

$$+ \sum_{m=1}^{\infty} m (G_{k+m}G_m + H_{k+m}H_m) \exp [-(\alpha_{k+m} + \alpha_m - \alpha_k)(r-r_o)] \qquad (8)$$

$$\left. - \sum_{m=k+1}^{\infty} m (G_{m-k}G_m + H_{m-k}H_m) \exp [-(\alpha_{m-k} + \alpha_m - \alpha_k)(r-r_o)] \right\}$$

and

$$\frac{\partial H_k}{\partial r} = \frac{b\omega_o}{2} (r_o/r)^a \left\{ \sum_{m=1}^{k-1} m (H_{k-m}G_m + G_{k-m}H_m) \exp [-(\alpha_{k-m} + \alpha_m - \alpha_k)(r-r_o)] \right.$$

$$+ \sum_{m=1}^{\infty} m(H_{k+m}G_m - G_{k+m}H_m) \exp [-(\alpha_{k+m} + \alpha_m - \alpha_k)(r-r_o)] \qquad (9)$$

$$\left. + \sum_{m=k+1}^{\infty} m(H_{m-k}G_m - G_{m-k}H_m) \exp [-(\alpha_{m-k} + \alpha_m - \alpha_k)(r-r_o)] \right\}.$$

4

The first sum on the right-hand side of Eqs. (8) and (9) represents contributions to the kth harmonic due to the interaction of lower harmonics producing a sum frequency component at the kth harmonic. The second sum in the equations corresponds to the interaction of higher harmonics producing a difference frequency component at the kth harmonic. The last sum in the equations represents the loss to the kth harmonic due to its interaction with all of the harmonics.

Combining the last two sums in Eqs. (8) and (9) yields

$$
\frac{\partial G_k}{\partial r} = \frac{b\omega_o}{2} (r_o/r)^a \left\{ \sum_{m=1}^{k-1} m (G_{k-m}G_m - H_{k-m}H_m) \exp[-(\alpha_{k-m}+\alpha_m-\alpha_k)(r-r_o)] \right.
$$
$$
\left. -k \sum_{m=1}^{\infty} (G_{k+m}G_m + H_{k+m}H_m) \exp[-(\alpha_{k+m}+\alpha_m-\alpha_k)(r-r_o)] \right\}
\tag{10}
$$

and

$$
\frac{\partial H_k}{\partial r} = \frac{b\omega_o}{2} (r_o/r)^a \left\{ \sum_{m=1}^{k-1} m (H_{k-m}G_m + H_m G_{k-m}) \exp[-(\alpha_{k-m}+\alpha_m-\alpha_k)(r-r_o)] \right.
$$
$$
\left. +k \sum_{m=1}^{\infty} (H_m G_{k+m} - H_{k+m}G_m) \exp[-(\alpha_{k+m}+\alpha_m-\alpha_k)(r-r_o)] \right\}.
\tag{11}
$$

Equations (10) and (11) are the coupled nonlinear equations that are numerically integrated to obtain the harmonic amplitudes G_k and H_k as a function of the spatial coordinate r.

II. RELATIVE IMPORTANCE OF LINEAR ABSORPTION AND NONLINEAR EFFECTS

The Goldberg number Γ is defined as

$$
\Gamma = \frac{1}{\alpha \ell}
\tag{12}
$$

where α is the absorption coefficient and ℓ is the discontinuity distance, the point at which the waveform would shock if linear absorption were absent. The discontinuity distance is geometry dependent and for initially sinusoidal waves is given by

$$\ell_p = \frac{c_o^2}{\beta U_o \omega_o} \quad \text{(plane wave),} \tag{13}$$

$$\ell_c = r_o + \frac{c_o^2}{\beta U_o \omega_o} + \frac{c_o^4}{4 r_o \beta^2 U_o^2 \omega_o^2} \quad \text{(cylindrical wave),} \tag{14}$$

and

$$\ell_s = r_o \exp(c_o^2 / \beta U_o \omega_o r_o) \quad \text{(spherical wave).} \tag{15}$$

If the Goldberg number is greater than unity, the nonlinear effect becomes important and shocks are likely. In this case, as the waveform approaches the discontinuity distance, nonlinear effects dominate the loss due to linear absorption. The amplitudes of all harmonics above the fundamental increase at the expense of the fundamental. After the discontinuity distance is reached, however, linear absorption plays an increasingly larger role and eventually the amplitudes of all the harmonics decrease with distance.

The use of the Goldberg number is important in deciding on the number of harmonics to retain in the calculation. If the Goldberg number is small compared to unity, then the nonlinear effect is small and the waveform is not going to shock. In this case, a small number of harmonics will adequately describe the waveform at any position. However, if the Goldberg number is large, then shocks are likely and a large number of harmonics must be retained in the calculation. A discussion of the relative error associated with the number of harmonics retained in the calculation is found in Section IV.E.

III. NUMERICAL IMPLEMENTATION

A. Series Truncation

In order to numerically integrate Eqs. (10) and (11) the infinite
series on the right-hand side of each equation must be truncated in such a
manner that no instability is introduced into the algorithm. This problem can
best be examined by assuming that the phase of the initial waveform is such
that all the amplitude coefficients H_k are zero and that j harmonics are retained
in the calculation. This results in Eq. (11) vanishing and Eq. (10) reducing
to

$$\frac{\partial G_k}{\partial r} = -\frac{b\omega_o}{2}(r_o/r)^a\left\{\sum_{m=1}^{k-1} m\, G_{k-m}G_m \exp\left[-(\alpha_{k-m}+\alpha_m-\alpha_k)(r-r_o)\right]\right.$$

$$\left. -k\sum_{m=1}^{j-k} G_{k+m}G_m \exp\left[-(\alpha_{k+m}+\alpha_m-\alpha_k)(r-r_o)\right]\right\}. \tag{16}$$

The simple truncation used in obtaining Eq. (16) is insufficient when
an attempt is made to examine the propagation of the waveform beyond the dis-
continuity distance. In calculating the propagation, the flow of energy from
lower to higher harmonics stops with the last harmonic retained in the series.
This is obvious from the fact that the second series in Eq. (16) vanishes for
the jth harmonic. Thus the use of simple truncation eliminates the primary
nonlinear energy-loss mechanism of the last (j^{th}) harmonic. The harmonics
preceding the last are affected in a similar, but less severe, manner. The
calculated values for the last harmonics become abnormally large relative to
the lower harmonics. Being "too large", these harmonics then cause an abnormal
growth of the next lower harmonics so that eventually even the lowest harmonics
are significantly in error. This instability is circumvented in the program by
artificially increasing the loss of the last few (and least significant) harmonics
by requiring that their amplitude never exceed the amplitude of the next lower
harmonic.

B. Integration Method

Equations (10) and (11) are stepwise numerically integrated by the first-order Runge-Kutta method to obtain the amplitudes of the harmonics at progressively increasing distances. This method, as applied to the numerical solution of the problem

$$G_k' = \frac{\partial G_k}{\partial r} = R_k(r, G, H) \tag{17}$$

and

$$H_k' = \frac{\partial H_k}{\partial r} = S_k(r, G, H) \tag{18}$$

yields the amplitudes at the $N+1^{th}$ step as

$$G_k(N+1) = G_k(N) + \frac{h}{2}\left[G_k'(N) + G_k'(N+1)\right] \tag{19}$$

and

$$H_k(N+1) = H_k(N) + \frac{h}{2}\left[H_k'(N) + H_k'(N+1)\right], \tag{20}$$

where

$$G_k'(N) = R_k\left[r(N), G(N), H(N)\right], \tag{21}$$

$$H_k'(N) = S_k\left[r(N), G(N), H(N)\right], \tag{22}$$

$$G_k'(N+1) = R_k\left[r(N+1), G(N) + hG'(N), H(N) + hH'(N)\right], \tag{23}$$

$$H_k'(N+1) = S_k\left[r(N+1), G(N) + hG'(N), H(N) + hH'(N)\right], \tag{24}$$

and h is the incremental step size [9]. This procedure has the advantage of
not requiring the calculation of any derivatives of R_k and S_k as would be
necessary in a Taylor-series expansion. The procedure also allows the step
size to be easily changed at any point in the calculation. The only disadvantage
is that the right-hand sides of Eqs. (17) and (18) must be evaluated twice
at each step.

C. Variable Step Size

In calculating the propagation of a finite amplitude wave it is desirable
to use the largest step size that will produce accurate results. During the
initial portion of the propagation, when the waveform is undergoing its most
rapid change due to nonlinear effects, a small step size is required. After
the discontinuity distance is reached, however, the nonlinear effects become
less pronounced and the step size can be increased. In order to minimize the
running time of the computer program, both under these circumstances and on
occasions when an overly conservative initial step size has been chosen,
a variable step size feature is incorporated.

This feature doubles the step size whenever the average percentage
change in the amplitude components $|\Delta G_j/G_j|$ and $|\Delta H_j/H_j|$, j= 1, 2, . . ., k,
over the previous step is below some arbitrary value ϵ. The integer k is the
number of harmonics printed in the output and is generally less than the number
of harmonics retained in the calculation. There are two methods for controlling
the doubling of the step size. Either the internal value ϵ may be modified or the
number of harmonics printed out may be changed.

The harmonics are printed out at fixed distance intervals (print out
distance interval = specified integer × initial step size). When the step
size is doubled, it is unlikely that the harmonics will be calculated at positions
coinciding with the print-out distance. The program circumvents this problem
by linearly interpolating the output from the calculated values.

9

D. Renormalization

One of the standard programing problems associated with numerical solutions is the limited exponent range of computers. The Advanced Scientific Computer (ASC) at the Naval Research Laboratory (NRL), for which this program was written, has an exponent range of -76 to $+76$. The terms most likely to exceed this range are the exponentials in Eqs. (10) and (11).

To illustrate this problem the exponentials in Eqs. (10) and (11) are examined for the case of an omega-squared frequency dependence of the absorption coefficients (fresh water). The exponential in the second series on the right-hand sides of Eqs. (10) and (11) may then be written in the form

$$\exp\left[-(\alpha_{k+m}+\alpha_m-\alpha_k)(r-r_0)\right] = \exp\left[-2m(k+m)\alpha_1(r-r_0)\right], \qquad (25)$$

where α_1 is the absorption coefficient of the fundamental. Since m and k are positive integers, the exponential tends to zero with increasing r. This causes no problem if the computer is told to set underflow to zero (the error associated with setting numbers smaller than 10^{-76} to zero is negligible).

The exponential in the first series on the right-hand sides of Eqs. (10) and (11) may be written as

$$\exp\left[-(\alpha_{k+m}+\alpha_m-\alpha_k)(r-r_0)\right] = \exp\left[+2m(k-m)\alpha_1(r-r_0)\right]. \qquad (26)$$

Since m is always less than k in the first series, this exponential may exceed the upper bound of the exponent range. As an example, when 50 harmonics are retained in the calculation of a 100-kHz spherical wave in fresh water ($\alpha_1 = 2.38 \times 10^{-4}$ N/m), the exponent range of $+76$ is exceeded at $r = 255$ m. This distance is totally insufficient to examine the asymptotic decay of the spherical waves.

10

A simple method for circumventing this problem is to renormalize the waveform after each step. In this procedure the source position r_0 is changed after each step and set equal to the current position r. This limits the size of the distance term in the exponentials to h, the step size, and merely requires that the amplitudes be transformed as

$$A_k \longrightarrow A_k \; [R_0 \, / \, (R_0+h)] \; \exp \; (-\alpha_k h). \tag{27}$$

It has the additional advantage of not requiring the calculation of the exponentials at each step since they do not change and can be stored. If the step size doubles, the stored values are simply squared.

IV. DESCRIPTION OF COMPUTER PROGRAM FAW

The computer program FAW, which is listed in Appendix A, is written in universal FORTRAN and should run on any computer accepting this language. However, the program has been specifically written to take advantage of the vectorizing capability of the ASC at NRL.

While the use of the vectorizing option on the ASC greatly reduces the running time of the program, it does not allow the use of variable ranges on nested DO loops. This constraint results in FAW running inefficiently on computers that do not have vectorizing capability. Therefore, it may be necessary to modify FAW for use on a nonvectorizing computer.

The DO loops, which are modified in FAW for the vectorizing process, are the loops associated with the two series on the right-hand sides of Eqs. (10) and (11) (lines 243 to 256 and 320 to 333 in FAW). The first series on the right-hand sides of Eqs. (10) and (11) has an upper limit of k-1 for the kth harmonic. The second series is truncated to an upper limit of j-k for the kth harmonic when j harmonics are retained in the calculation. Since both of these limits are a function of the harmonic increment being calculated, the range of the inner DO loop is not a constant. However, the ASC runs quicker

11

if the loops are vectorized, and to do this the ranges of the two inner loops have been set equal to J-1. This results in extraneous terms being calculated and increases the running time on nonvectorizing computers.

The extraneous terms do not contribute to the calculation and are all set equal to zero in the DO loops. This is accomplished by generating a matrix for each DO loop whose elements are zero for the extraneous terms and whose nonzero elements are the exponential terms in Eqs. (10) and (11). As explained in the section on renormalization, this matrix need only be calculated once and is then used at each step.

The remainder of this section describes FAW. Included are a listing of the significant FORTRAN variable names and descriptions of the major computation blocks in FAW followed by a discussion of the parameter input, printed output, number of harmonics to retain in calculation, and initial step size.

A. Significant FORTRAN Variable Names

The significant FORTRAN variable names in FAW are as follows:

A   Geometrical spreading factor
                    = 0 (plane waves),
                    = 1/2 (cylindrical waves),
                    = 1 (spherical waves).

ALPHA   Vector whose I^{th} element ALPHA (I) is the absorption coefficient of the Ith harmonic.

B   The constant $b = \beta/c_o^2$ in Eqs. (10) and (11).

BETA   Coefficient of nonlinearity.

12

C   Small signal sound speed c_o.

DG   Vector containing the differential change of the amplitude
                  coefficients G. The element DG(I) contains the differential
                  change $hG_I'(N)$ at the Nth step as given by Eq. (21).

DG1   Vector whose I^{th} element DG1(I) contains the differential
                  change $hG_I'(N+1)$ at the $N+1^{th}$ step as given by Eq. (23).

DH   Vector containing the differential change of the amplitude
                  coefficients H. The element DH(I) contains the differential
                  change $hH_I'(N)$ at the Nth step as given by Eq. (22).

DH1   Vector whose I^{th} element DH1(I) contains the differential
                  change $hH'(N+1)$ at the $N+1^{th}$ step as given by Eq. (24).

DX   Current step size.

DXI   Initial step size.

E   Normalization constant. The output is normalized to the
                  constant E, which is an input parameter and generally set
                  equal to the initial amplitude of the fundamental.

F1   Factor from Eqs.(10) and (11) equal to $(r_o/r)^a$.

FREQ   Frequency of fundamental.

G   Vector whose I^{th} element contains the amplitude G(I) of
                  the sine component of the Ith harmonic.

GH   Vector whose I^{th} element GH(I) = exp [-ALPHA (I) * DX].

13

GX   Array whose elements are all zero that is used as a buffer for the working array G2. In the DO loops which calculate DG, DG1, DH, and DH1 the ranges of the loops have been written to take advantage of vectorization. This results in negative indices for some of the G2 elements in the calculation. The GX array is placed before the G2 array to prevent incorrect results.

G2   Array whose I^{th} element first contains G(I) and in later calculations contains G(I) + DG(I) as required by the Runge-Kutta method.

H   Same as G for the cosine elements.

HX   Same as GX but placed in front of H2.

H2   Same as G2 for the cosine elements.

IAF   Input parameter equal to either zero, if only the absorption coefficient for the fundamental is input and an omega-squared dependence for the harmonic absorption coefficients is used, or one if the absorption coefficient is input for each harmonic retained in the calculations.

IP   Print-out interval = IP * DX.

J   Number of harmonics retained in the calculations.

K   Number of input G coefficients.

KI   Number of input H coefficients.

14

L   Number of externally supplied absorption coefficients
                used to modify the omega-squared dependent coefficients.
                Used only if IAF = 0 and the omega-squared dependence is to
                be modified.

NI   Number of harmonics printed to output.

R   Spatial coordinate.

RMAX   Maximum distance that propagation is to be calculated.

RN   Source position. Modified after each step. Set equal to
                the present position R as described in section on renormal-
                ization.

RO   Initial source position.

X1   Doubly dimensioned array whose elements X1 (M, N) are unity
                if X11 (M, N) is nonzero and zero otherwise.

X11   Array whose elements X11 (M, N) = exp ((ALPHA(M) -
                ALPHA(N) - ALPHA (M-N)) * DX) for M = 2 to J and N = 1
                to M. All other elements are zero.

X2   Doubly dimensioned array whose elements X2 (M, N) are unity
                if X22 (M, N) is nonzero and zero otherwise.

X22   Array whose elements X22 (M, N) = exp ((ALPHA(M) -
                ALPHA(N) - ALPHA (M+N)) * DX) for M = 1 to J-1 and
                N = 1 to J-M. All other elements are zero.

B. Major Computation Blocks in FAW

Descriptions of the major computation blocks in FAW are as follows:

Computation Block	Line Number	
	From	To
Initialize all arrays to zero.	29	44
Read first fourteen data cards and print data to output.	51	138
Obtain absorption coefficients.	142	179
Read input waveform and print waveform to output.	183	199
Calculate matrix elements of X1, X11, X2, and X22 and elements of vector GH.	204	221
Calculate DG and DH, first derivative of Runge-Kutta method.	243	256
Calculate new amplitudes for second derivative.	260	265
Find last five non-zero harmonic amplitudes and modify them, if necessary, to insure that they form a non-increasing sequence.	289	308
Calculate DG1 and DH1, second derivative of Runge-Kutta method.	320	333

16

Computation Block	Line Number	
	From	To
Calculate new amplitudes for current position.	337	340
Find last five non-zero harmonic amplitudes and modify them, if necessary, to insure that they form a non-increasing sequence.	344	382
Check position for output.	386	386
Interpolate output if output position does not coincide with position of calculated amplitudes.	390	404
Standard output (no interpolation necessary).	405	408
Print output.	409	418
Check step size (if incremental change is small, then double step size).	422	431
Modify waveform for renormalization.	435	438
Double step size.	443	454
Next step.	456	

17

C. Parameter Input

The input consists of a series of data cards:

Data Card 1:   Format D17.10 — FREQ, the frequency of the fundamental.

Data Card 2:   Format D17.10 — C, the small signal sound speed in meters per second.

Data Card 3:   Format D17.10 — BETA, the coefficient of nonlinearity.

Data Card 4:   Format D17.10 — E, the normalization constant.

Data Card 5:   Format D17.10 — DXI, the initial step size in meters.

Data Card 6:   Format D17.10 — RMAX, the maximum propagation distance in meters.

Data Card 7:   Format D17.10 — RO, the source size, in meters, for cylindrical and spherical waves. If a plane wave is being calculated, RO is not used but a value must be entered.

Data Card 8:   Format F5.2 — A, the geometrical spreading factor.
    A = 0 (plane waves)
      = 1/2 (cylindrical waves)
      = 1 (spherical waves).

Data Card 9:   Format I4 — IAF, the absorption flag. IAF = 0 if the omega-squared dependence for the absorption coefficients is being used. IAF = 1 if all absorption coefficients are entered on data cards.

Data Card 10: Format I4 — J, the number of harmonics retained in the calculation.

Data Card 11:   Format I4 – K, the number of initial G coefficients entered.

Data Card 12:   Format I4 – KI, the number of initial H coefficients entered.

Data Card 13:   Format I4 – NI, the number of harmonics printed to output.

Data Card 14:   Format I8 – IP, the integer multiplicative factor of
                DXI which gives the print-out interval.

If IAF = 0

Data Card 15:   Format D17.10 – ALPHA (1), the absorption coefficient
                of the fundamental in nepers per meter.

Data Card 16:   Format I4 – L, the number of harmonics being modified
                from the omega-squared dependence.

Next L Cards: Format I4, D17.10 – These cards contain an integer, right
                justified in the first four spaces on the card, specifying the
                number of the harmonic followed by the absorption coefficient
                for that harmonic.

If IAF = 1

Data Card 15 to 14+J: Format D17.10 – These cards contain the J absorption
                coefficients in order in nepers per meter.

Next K Cards: Format I4, D17.10 – These cards contain a right justified
                integer, in the first four spaces, specifying the number of
                the harmonic followed by the G amplitude coefficient of that
                harmonic in meters per second.

Next KI Cards: Format I4, D17.10 – These cards contain a right justified
                integer, in the first four spaces, specifying the number of
                the harmonic followed by the H amplitude coefficient of
                that harmonic in meters per second.

D. Printed Output

A sample output from FAW is shown in Appendix B. The first portion of the output contains a listing of all input parameters. This allows the input parameters to be checked for errors and is useful for future reference. The remaining portion contains the calculated output.

The distance in meters is printed at each output interval followed by two columns of numbers. The columns are labeled and contain the amplitudes of the sine and cosine components divided by the normalization constant F. The number of terms output at each interval is NI, which is a user-specified parameter. In addition, the step size DX in meters, is printed each time the step size is doubled. This gives the user useful information on the effects of varying the step size doubling parameter ϵ and the number of harmonics printed out NI.

E. Number of Harmonics

A sufficient number of harmonics must be retained in the calculation to insure a negligibly small error in the highest harmonic of interest. In order to obtain some measure of the required number of harmonics, the algorithm was used to calculate the harmonic content of an initially pure sinusoidal plane wave with a variety of harmonics being retained in the calculation. The frequency of the fundamental was 2.5 MHz, and the initial pressure amplitude was 3 atmospheres, which gives a discontinuity distance of 21 cm. Table I lists the percentage deviations of the resulting amplitudes of the first five harmonics, at the discontinuity distance, from the values obtained when 40 harmonics were retained. As is obvious from the table, the required number of harmonics to retain depends on the harmonic of interest and the allowable error. If the fundamental is the only harmonic of interest, one need retain no more than seven harmonics in the calculation. On the other hand, an accurate value for the fifth harmonic may require twenty or more harmonics to be retained in the calculation.

Table 1 - Percentage deviation in the first five harmonics
for various numbers of retained harmonics N

HARMONIC	7	8	9	10	15	20	25	30
1st	0.03	0.02	0.02	0.02	0.004	0.0003	0.00002	0.00000
2nd	0.20	0.20	0.10	0.10	0.020	0.0020	0.00030	0.00003
3rd	0.80	0.60	0.50	0.50	0.080	0.0090	0.00100	0.00010
4th	2.40	1.80	1.60	1.50	0.200	0.0200	0.00300	0.00040
5th	4.40	3.80	3.50	3.40	0.500	0.0500	0.00700	0.00080

F. Step Size

With the ability of the program to double the step size after each step,
it is best to choose the initial step size conservatively and let the program
find the best value. In order to determine a conservative initial value for
the step size, the choice of step size was investigated for the same 2.5-MHz
plane-wave case used in the previous section. The step size was not allowed
to double and various step sizes from 1/200 to 1/10 of the discontinuity
distance were used. Table II lists the percentage deviations of the first
five harmonic amplitudes from the values obtained when the step size was 1/200
of the discontinuity distance. The amplitudes were those at the discontinuity
distance, and forty harmonics were retained in the calculations. The table
indicates that a step size of 1/10 of the discontinuity distance will yield
results that are accurate to within the normal experimental error.

Table II – Percentage deviation in the first five harmonics
for various step sizes

(σ = discontinuity distance)

STEP SIZE HARMONIC	1/100 σ	1/50 σ	1/20 σ	1/10 σ
1st	0.0005	0.0030	0.0200	0.0600
2nd	0.0001	0.0050	0.0300	0.1000
3rd	0.0020	0.0100	0.0900	0.4000
4th	0.0070	0.0400	0.2500	1.1000
5th	0.0100	0.0700	0.5000	2.0000

V. COMPARISON WITH PHENOMENOLOGICAL MODEL

As a test of their validity, Eqs. (10) and (11) were used to compute the
harmonic content of an initially pure sinusoidal 300-Hz plane wave. An omega-
squared frequency dependence of the linear absorption terms was assumed with
the absorption of the fundamental being 1.12×10^{-6} Np/m. The initial pressure
was 1 atmosphere. This problem was also solved using a computer algorithm
[6] based on the phenomenological model of Fox and Wallace [7]. Figure (1)
illustrates the agreement between the results obtained using the algorithm
presented in this paper, shown as solid curves, and the results obtained using
the phenomenological model, shown as dots. Although only the first four
harmonics are illustrated in this figure, the agreement was equally as good
for higher harmonics.

Fig. 1 - Calculated harmonic behavior of an initially pure sinusoidal plane wave (ν = 300 Hz, α = 1.12 x 10^{-6} N/m, and P_o = 1 atm.) by ———— frequency domain algorithm, • phenomenological model.

23

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APPENDIX A

COMPUTER PROGRAM FAW

```
 1 C
 2 C
 3 C
 4 C       THIS PROGRAM STEPWISE CALCULATES THE HARMONIC CONTENT
 5 C       OF A FINITE AMPLITUDE WAVE AS A FUNCTION OF POSITION.
 6 C       THE INITIAL WAVEFORM MAY HAVE ANY HARMONIC CONTENT
 7 C       AND ARBITRARY PHASE. THE ABSORPTION COEFFICIENTS MAY
 8 C       BE EXTERNALLY SUPPLIED OR AN INTERNAL OMEGA SQUARED
 9 C       ALGORITHM IS SUPPLIED. THE INTERNAL ALGORITHM ALLOWS
10 C       MODIFICATION OF ANY OF THE OMEGA SQUARED COEFFICIENTS.
11 C       THE NUMERICAL INTEGRATION USES THE RUNGE-KUTTA METHOD
12 C       AND THE STEP SIZE WILL AUTOMATICALLY INCREASE WHEN THE
13 C       CHANGE OVER AN INTERVAL IS SMALL.
14 C
15 C
16 C
17         DIMENSION G(50),GX(50),G2(100),H(50),HX(50),H2(100)
18         DIMENSION X1(50,50),X11(50,50),X2(50,50)
19         DIMENSION X22(50,50),XL(50),XL1(50),GH(50)
20         DIMENSION ALPHA(50),DG(50),DG1(50),DH(50),DH1(50)
21         DIMENSION KG(5),KH(5)
22         DOUBLE PRECISION G,GX,G2,H,HX,H2,X1,X11,X2,X22,XL,XL1
23         DOUBLE PRECISION GH,ALPHA,DG,DG1,DH,DH1,R,C,FREQ,BETA
24         DOUBLE PRECISION E,B,X,F1,DX,RMAX,RN,Z,Z1,DXI,PD
25         DOUBLE PRECISION Y,RO
26 C
27 C       INITIALIZE REGISTERS TO ZERO
28 C
29         DO 110 IB=1,100
30         G(IB)=0.DO
31         GX(IB)=0.DO
32         G2(IB)=0.DO
33         H(IB)=0.DO
34         HX(IB)=0.DO
35         H2(IB)=0.DO
36 110     CONTINUE
37         DO 130 IC=1,50
38           DO 120 ID=1,50
39           X1(IC,ID)=0.DO
40           X11(IC,ID)=0.DO
41           X2(IC,ID)=0.DO
42           X22(IC,ID)=0.DO
43 120       CONTINUE
44 130     CONTINUE
45 C
46 C       INPUT DATA
47 C
48 C
49 C       FREQ=FUNDAMENTAL FREQUENCY
50 C
51         READ(5,140)FREQ
```

```
52 140      FORMAT(D17.10)
53          PRINT 150,FREQ
54 150      FORMAT(5X,10HFREQUENCY=,D17.10)
55 C
56 C        C=SMALL SIGNAL SOUND SPEED
57 C
58          READ(5,140)C
59          PRINT 160,C
60 160      FORMAT(5X,12HSOUND SPEED=,D17.10)
61 C
62 C        BETA=COEFFICIENT OF NONLINEARITY(1+B/2A)
63 C
64          READ(5,140)BETA
65          PRINT 170,BETA
66 170      FORMAT(5X,5HBETA=,D17.10)
67 C
68 C        E=NORMALIZATION CONSTANT (OUTPUT IN DB RE(E))
69 C
70          READ(5,140)E
71          PRINT 180,E
72 180      FORMAT(5X,23HNORMALIZATION CONSTANT=,D17.10)
73 C
74 C        DXI=INITIAL STEP SIZE
75 C
76          READ(5,140)DXI
77          PRINT 190,DXI
78 190      FORMAT(5X,18HINITIAL STEP SIZE=,D17.10)
79 C
80 C        RMAX=MAXIMUM PROPAGATION DISTANCE
81 C
82          READ(5,140)RMAX
83          PRINT 200,RMAX
84 200      FORMAT(5X,17HMAXIMUM DISTANCE=,D17.10)
85 C
86 C        RO=SOURCE SIZE(INITIAL POSITION FOR GEOMETRICAL SPREADING)
87 C
88          READ(5,140)RO
89          PRINT 210,RO
90 210      FORMAT(5X,12HSOURCE SIZE=,D17.10)
91 C
92 C        A=SPREADING FACTOR
93 C         =0 (PLANE WAVES)
94 C         =1/2 (CYLINDRICAL WAVES)
95 C         =1 (SPHERICAL WAVES)
96 C
97          READ(5,220)A
98 220      FORMAT(F5.2)
99          PRINT 230,A
100 230     FORMAT(5X,17HSPREADING FACTOR=,F5.2)
101 C
102 C       IAF- FLAG(ABSORPTION COEFFICIENTS)
```

26.

```
103 C          =0 (OMEGA SQUARED DEPENDENCE)
104 C          =1 (EXTERNALLY SUPPLIED)
105 C
106            READ(5,240)IAF
107 240        FORMAT(I4)
108            PRINT 250,IAF
109 250        FORMAT(5X,16HABSORPTION FLAG=,I4)
110 C
111 C      J=NUMBER OF HARMONICS RETAINED IN CALCULATION
112 C
113            READ(5,240)J
114            PRINT 260,J
115 260        FORMAT(5X,20HNUMBER OF HARMONICS=,I4)
116 C
117 C      K=NUMBER OF INITIAL G COEFFICIENTS
118 C      KI=NUMBER OF INITIAL H COEFFICIENTS
119 C
120            READ(5,240)K
121            READ(5,240)KI
122            PRINT 270,K
123 270        FORMAT(5X,23HINITIAL G COEFFICIENTS=,I4)
124            PRINT 280,KI
125 280        FORMAT(5X,23HINITIAL H COEFFICIENTS=,I4)
126 C
127 C      NI=NUMBER OF HARMONICS PRINTED OUT
128 C
129            READ(5,240)NI
130            PRINT 290,NI
131 290        FORMAT(5X,27HNUMBER OF HARMONICS OUTPUT=,I4)
132 C
133 C      IP-PRINT OUT INTERVAL=IP*DX
134 C
135            READ(5,300)IP
136 300        FORMAT(I8)
137            PRINT 310,IP
138 310        FORMAT(5X,19HPRINT OUT INTERVAL=,I8)
139 C
140 C      OBTAIN ABSORPTION COEFFICIENTS
141 C
142            IF(IAF)1110,320,400
143 C
144 C      OMEGA SQUARED DEPENDENCE
145 C
146 320        READ(5,140)ALPHA(1)
147            PRINT 330,ALPHA(1)
148 330        FORMAT(5X,9HALPHA(1)=,D17.10)
149            DO 340 IE=2,J
150            ALPHA(IE)=IE*IE*ALPHA(1)
151 340        CONTINUE
152 C
153 C      L=NUMBER OF ABSORPTION COEFFICIENTS BEING MODIFIED
```

```
154 C          FROM OMEGA SQUARED DEPENDENCE
155 C
156            READ(5,240)L
157            PRINT 350,L
158 350        FORMAT(5X,43HNUMBER OF MODIFIED ABSORPTION COEFFICIENTS=,I4)
159            IF(L)1110,440,360
160 360        PRINT 370
161 370        FORMAT(5X,33HMODIFIED ABSORPTION COEFFICIENTS.)
162            DO 390 IG=1,L
163            READ(5,380)N,X
164 380        FORMAT(I4,D17.10)
165            PRINT 380,N,X
166            ALPHA(N)=X
167 390        CONTINUE
168            GO TO 440
169 C
170 C          INPUT ABSORPTION COEFFICIENTS
171 C
172 400        PRINT 410
173 410        FORMAT(5X,23HABSORPTION COEFFICIENTS)
174            DO 430 IH=1,J
175            READ(5,140)X
176            ALPHA(IH)=X
177            PRINT 420,IH,ALPHA(IH)
178 420        FORMAT(1X,6HALPHA(,I3,2H)=,D17.10)
179 430        CONTINUE
180 C
181 C          READ INPUT WAVEFORM
182 C
183 440        CONTINUE
184            IF(K)1110,490,450
185 450        PRINT 460
186 460        FORMAT(5X,14HINPUT WAVEFORM)
187            DO 480 IJ=1,K
188            READ(5,380)N,X
189            G(N)=X
190            PRINT 470,N,G(N)
191 470        FORMAT(1X,2HG(,I3,2H)=,D17.10)
192 480        CONTINUE
193 490        IF(K1)1110,530,500
194 500        DO 520 IK=1,K1
195            READ(5,380)N,X
196            H(N)=X
197            PRINT 510,N,H(N)
198 510        FORMAT(1X,2HH(,I3,2H)=,D17.10)
199 520        CONTINUE
200 530        B=3.1415926536*BETA*FREQ/(C*C)
201 C
202 C          CALCULATE MATRIX ELEMENTS
203 C
204            DO 550 IL=2,J
```

```
205          IL1=IL-1
206             DO 540 IM=1,IL1
207             X11(IL,IM)=DEXP((ALPHA(IL)-ALPHA(IM)-ALPHA(IL-IM))*DXI)
208             X1(IL,IM)=1.D0
209 540         CONTINUE
210 550      CONTINUE
211          IFX=J-1
212          DO 570 IN=1,IFX
213          IN1=J-IN
214             DO 560 IP=1,IN1
215             X22(IN,IP)=DEXP((ALPHA(IN)-ALPHA(IP)-ALPHA(IN+IP))*DXI)
216             X2(IN,IP)=1.D0
217 560         CONTINUE
218 570      CONTINUE
219          DO 580 IQ=1,J
220          GH(IQ)=DEXP(-ALPHA(IQ)*DXI)
221 580      CONTINUE
222 C
223 C        SET COUNTER
224 C
225          RN=RO
226          R=RN
227          DX=DXI
228          PD=IF*DX
229          F1=1.D0
230 590      CB=DX*B
231 C
232 C        FILL ARRAYS
233 C
234          DO 600 IR=1,J
235          G2(IR)=G(IR)
236          H2(IR)=H(IR)
237          DG(IR)=0.D0
238          DH(IR)=0.D0
239 600      CONTINUE
240 C
241 C        ENTER LOOP FOR CALCULATING FIRST DERIVATIVE
242 C
243          DO 620 IS=2,J
244             DO 610 IT=1,IFX
245             CX=CB*IT*X1(IS,IT)
246             DG(IS)=CX*(G2(IS-IT)*G2(IT)-H2(IS-IT)*H2(IT))+DG(IS)
247             DH(IS)=CX*(H2(IS-IT)*G2(IT)+G2(IS-IT)*H2(IT))+DH(IS)
248 610         CONTINUE
249 620      CONTINUE
250          DO 640 IU=1,J
251             DO 630 IV=1,IFX
252             CX=CB*IU*X2(IU,IV)
253             DG(IU)=DG(IU)-CX*(G2(IU+IV)*G2(IV)+H2(IU+IV)*H2(IV))
254             DH(IU)=DH(IU)+CX*(G2(IU+IV)*H2(IV)-H2(IU+IV)*G2(IV))
255 630         CONTINUE
```

```
256 640     CONTINUE
257 C
258 C       CALCULATE NEW AMPLITUDES FOR SECOND DERIVATIVE
259 C
260         DO 650 IW=1,J
261         G2(IW)=G(IW)+DG(IW)
262         H2(IW)=H(IW)+DH(IW)
263         DG1(IW)=0.D0
264         DH1(IW)=0.D0
265 650     CONTINUE
266 C
267 C       FIND LAST FIVE NON-ZERO HARMONICS
268 C
269         DO 655 IW1=1,5
270         KG(IW1)=0
271         KH(IW1)=0
272 655     CONTINUE
273         DO 710 IX=1,J
274         IF(G2(IX))660,680,660
275 660         DO 670 IY=1,4
276             KG(IY)=KG(IY+1)
277 670         CONTINUE
278         KG(5)=IX
279 680     IF(H2(IX))690,710,690
280 690         DO 700 IZ=1,4
281             KH(IZ)=KH(IZ+1)
282 700         CONTINUE
283         KH(5)=IX
284 710     CONTINUE
285 C
286 C       INSURE THAT THE LAST FIVE HARMONICS ARE NOT
287 C       PROGRESSIVELY LARGER
288 C
289         DO 720 JA=1,4
290         JB=KG(JA)
291         IF(JB)712,720,712
292 712     Z=DABS(G2(JB))
293         JC=KG(JA+1)
294         IF(JC)714,720,714
295 714     Z1=DABS(G2(JC))
296         IF(Z1.LT.Z)GO TO 720
297         G2(JC)=0.95*G2(JC)*Z/Z1
298 720     CONTINUE
299         DO 730 JD=1,4
300         JE=KH(JD)
301         IF(JE)722,730,722
302 722     Z=DABS(H2(JE))
303         JF=KH(JD+1)
304         IF(JF)724,730,724
305 724     Z1=DABS(H2(JF))
306         IF(Z1.LT.Z)GO TO 730
```

```
307         H2(JF)=0.95*H2(JF)*Z/Z1
308  730    CONTINUE
309         R=R+DX
310         IF(A)1110,760,740
311  740    IF(A.LT.1.D0)GO TO 750
312         F1=RN/R
313         CB=DX*B*F1
314         GO TO 760
315  750    F1=(RN/R)**0.5
316         CB=DX*B*F1
317  C                .        .
318  C      ENTER LOOP FOR CALCULATING SECOND DERIVATIVE
319  C
320  760    DO 780 JG=2,J
321            DO 770 JH=1,IFX
322            CX=CB*JH*X11(JG,JH)
323            DG1(JG)=CX*(G2(JG-JH)*G2(JH)-H2(JG-JH)*H2(JH))+DG1(JG)
324            DH1(JG)=CX*(H2(JG-JH)*G2(JH)+G2(JG-JH)*H2(JH))+DH1(JG)
325  770      CONTINUE
326  780    CONTINUE
327         DO 800 JI=1,J
328            DO 790 JJ=1,IFX
329            CX=CB*JI*X22(JI,JJ)
330            DG1(JI)=DG1(JI)-CX*(G2(JI+JJ)*G2(JJ)+H2(JI+JJ)*H2(JJ))
331            DH1(JI)=DH1(JI)+CX*(G2(JI+JJ)*H2(JJ)-H2(JI+JJ)*G2(JJ))
332  790      CONTINUE
333  800    CONTINUE
334  C
335  C      CALCULATE NEW AMPLITUDES
336  C
337         DO 810 JK=1,J
338         G(JK)=G(JK)+0.5*(DG(JK)+DG1(JK))
339         H(JK)=H(JK)+0.5*(DH(JK)+DH1(JK))
340  810    CONTINUE
341  C
342  C      FIND LAST FIVE NON-ZERO HARMONICS
343  C
344         DO 820 JL=1,5
345         KG(JL)=0
346         KH(JL)=0
347  820    CONTINUE
348         DO 880 JM=1,J
349         IF(G(JM))830,850,830
350  830      DO 840 JN=1,4
351            KG(JN)=KG(JN+1)
352  840      CONTINUE
353         KG(5)=JM
354  850    IF(H(JM))860,880,860
355  860      DO 870 JO=1,4
356            KH(JO)=KH(JO+1)
357  870      CONTINUE
```

```
358          KH(5)=JM
359 830      CONTINUE
360 C
361 C        INSURE THAT THE LAST FIVE HARMONICS ARE NOT PROGRESSIVELY LARGER
362 C
363          DO 890 JP=1,4
364          JQ=KG(JP)
365          IF(JQ)882,890,882
366 882      Z=DABS(G(JQ))
367          JR=KG(JP+1)
368          IF(JR)884,890,884
369 884      Z1=DABS(G(JR))
370          IF(Z1.LT.Z)GO TO 890
371          G(JR)=0.95*G(JR)*Z/Z1
372 890      CONTINUE
373          DO 900 JS=1,4
374          JT=KH(JS)
375          IF(JT)892,900,892
376 892      Z=DABS(H(JT))
377          JU=KH(JS+1)
378          IF(JU)894,900,894
379 894      Z1=DABS(H(JU))
380          IF(Z1.LT.Z)GO TO 900
381          H(JU)=0.95*H(JU)*Z/Z1
382 900      CONTINUE
383 C
384 C        CHECK FOR OUTPUT DISTANCE
385 C
386 905      IF((R-RO)-PD)1010,950,910
387 C
388 C        INTERPOLATE OUTPUT
389 C
390 910      Y=DX-((R-RO)-PD)
391          DO 920 JV=1,NI
392          G2(JV)=G2(JV)-DG(JV)+0.5*(DG(JV)+DG1(JV))*Y/DX
393          H2(JV)=H2(JV)-DH(JV)+0.5*(DH(JV)+DH1(JV))*Y/DX
394 920      CONTINUE
395          X=F1
396          IF(A-0.5)940,925,930
397 925      X=(RN/(RN+Y))**0.5
398          GO TO 940
399 930      X=RN/(RN+Y)
400 940      DO 945 JV1=1,NI
401          XL(JV1)=X*G2(JV1)*DEXP(-ALPHA(JV1)*Y)/E
402          XL1(JV1)=X*H2(JV1)*DEXP(-ALPHA(JV1)*Y)/E
403 945      CONTINUE
404          GO TO 965
405 950      DO 960 JX=1,NI
406          XL(JX)=F1*G(JX)*GH(JX)/E
407          XL1(JX)=F1*H(JX)*GH(JX)/E
408 960      CONTINUE
```

```
409 965     PRINT 970,PD
410 970     FORMAT(5X,9HDISTANCE=,D17.10)
411         PRINT 980
412 980     FORMAT(5X,3HSIN,20X,3HCOS)
413         DO 1000 JY=1,NI
414         PRINT 990,JY,XL(JY),XL1(JY)
415 990     FORMAT(1X,I4,4X,D17.10,5X,D17.10)
416 1000    CONTINUE
417         PD=PD+IP*DXI
418         GO TO 905
419 C
420 C       CHECK STEP SIZE
421 C
422 1010    N=0
423         Z=0.D0
424         DO 1050 J1=1,NI
425         IF(G(J1))1020,1030,1020
426 1020    Z=Z+DABS(0.5*((DG(J1)+DG1(J1))/G(J1)))
427         N=N+1
428 1030    IF(H(J1))1040,1050,1040
429 1040    Z=Z+DABS(0.5*(DH(J1)+DH1(J1))/H(J1))
430         N=N+1
431 1050    CONTINUE
432 C
433 C       MODIFY WAVEFORM
434 C
435         DO 1060 J2=1,J
436         G(J2)=G(J2)*F1*GH(J2)
437         H(J2)=H(J2)*F1*GH(J2)
438 1060    CONTINUE
439         IF(Z.GT.(N*0.005))GO TO 1100
440 C
441 C       DOUBLE STEP SIZE
442 C
443         DX=2.0*DX
444         DO 1070 J2=1,J
445         GH(J2)=GH(J2)*GH(J2)
446 1070    CONTINUE
447         DO 1090 J3=1,J
448            DO 1080 J4=1,J
449            X11(J3,J4)=X11(J3,J4)*X11(J3,J4)
450            X22(J3,J4)=X22(J3,J4)*X22(J3,J4)
451 1080       CONTINUE
452 1090    CONTINUE
453         PRINT 1106,DX
454 1106    FORMAT(1X,3HDX=,D17.10)
455 1100    RN=R
456         IF(RMAX-(R-R0))1110,590,590
457 1110    STOP
458         END
```

APPENDIX B

SAMPLE OUTPUT FROM FAW

```
FREQUENCY= 0.3000000000D 03
SOUND SPEED= 0.1500000000D 04
BETA= 0.3500000000D 01
NORMALIZATION CONSTANT= 0.6670000000D-01
INITIAL STEP SIZE= 0.1000000000D 02
MAXIMUM DISTANCE= 0.1000000000D 05
SOURCE SIZE= 0.0000000000D 00
SPREADING FACTOR= 0.00
ABSORPTION FLAG=    0
NUMBER OF HARMONICS=   40
INITIAL G COEFFICIENTS=    1
INITIAL H COEFFICIENTS=    0
NUMBER OF HARMONICS OUTPUT=    5
PRINT OUT INTERVAL=       100
ALPHA(1)= 0.1120000000D-05
NUMBER OF MODIFIED ABSORPTION COEFFICIENTS=    0
INPUT WAVEFORM
G(   1)= 0.6670000000D-01
DISTANCE= 0.1000000000D 04
SIN                     COS
1      0.9941231144D 00      0.0000000000D 00
2      0.9622622120D-01      0.0000000000D 00
3      0.1394984526D-01      0.0000000000D 00
4      0.2394449333D-02      0.0000000000D 00
5      0.4511965196D-03      0.0000000000D 00
DISTANCE= 0.2000000000D 04
SIN                     COS
1      0.9788882494D 00      0.0000000000D 00
2      0.1845999987D 00      0.0000000000D 00
3      0.5197039094D-01      0.0000000000D 00
4      0.1729678606D-01      0.0000000000D 00
5      0.6313700593D-02      0.0000000000D 00
DISTANCE= 0.3000000000D 04
SIN                     COS
1      0.9546672934D 00      0.0000000000D 00
2      0.2587699767D 00      0.0000000000D 00
3      0.1041698315D 00      0.0000000000D 00
4      0.4945203382D-01      0.0000000000D 00
5      0.2570994911D-01      0.0000000000D 00
DX= 0.2000000000D 02
DISTANCE= 0.4000000000D 04
SIN                     COS
1      0.9220214992D 00      0.0000000000D 00
2      0.3138414940D 00      0.0000000000D 00
3      0.1574155764D 00      0.0000000000D 00
4      0.9275848905D-01      0.0000000000D 00
5      0.5972644773D-01      0.0000000000D 00
DX= 0.4000000000D 02
```

```
    DISTANCE= 0.5000000000D 04
      SIN                    COS
  1       0.8817469738D 00      0.0000000000D 00
  2       0.3469048248D 00      0.0000000000D 00
  3       0.1988133777D 00      0.0000000000D 00
  4       0.1330635772D 00      0.0000000000D 00
  5       0.9695954537D-01      0.0000000000D 00
DX= 0.8000000000D 02
    DISTANCE= 0.6000000000D 04
      SIN                    COS
  1       0.8353073629D 00      0.0000000000D 00
  2       0.3581480161D 00      0.0000000000D 00
  3       0.2202056808D 00      0.0000000000D 00
  4       0.1567515441D 00      0.0000000000D 00
  5       0.1208128685D 00      0.0000000000D 00
DX= 0.1600000000D 03
    DISTANCE= 0.7000000000D 04
      SIN                    COS
  1       0.7863635495D 00      0.0000000000D 00
  2       0.3538678457D 00      0.0000000000D 00
  3       0.2247481885D 00      0.0000000000D 00
  4       0.1638955398D 00      0.0000000000D 00
  5       0.1287869223D 00      0.0000000000D 00
DX= 0.3200000000D 03
    DISTANCE= 0.8000000000D 04
      SIN                    COS
  1       0.7389351302D 00      0.0000000000D 00
  2       0.3423198787D 00      0.0000000000D 00
  3       0.2210334863D 00      0.0000000000D 00
  4       0.1629226548D 00      0.0000000000D 00
  5       0.1290571601D 00      0.0000000000D 00
    DISTANCE= 0.9000000000D 04
      SIN                    COS
  1       0.6947357383D 00      0.0000000000D 00
  2       0.3279410902D 00      0.0000000000D 00
  3       0.2137387066D 00      0 0000000000D 00
  4       0.1583862311D 00      0.0000000000D 00
  5       0.1259628330D 00      0.0000000000D 00
    DISTANCE= 0.1000000000D 05
      SIN                    COS
  1       0.6542737913D 00      0.0000000000D 00
  2       0.3128153020D 00      0.0000000000D 00
  3       0.2050573568D 00      0.0000000000D 00
  4       0.1523892258D 00      0.0000000000D 00
  5       0.1214645359D 00      0.0000000000D 00
```